

INTRODUCTION TO THE MODERN THEORY OF DYNAMICAL SYSTEMS

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ABSTRACT. These are corrigenda for the book Introduction to the Modern Theory of Dynamical Systems.

1. THE HADAMARD–PERRON THEOREM

For the second printing several pages were entirely rewritten. These are available separately. They contain the smoothness argument in the proof of the Hadamard–Perron Theorem.

2. CHANGES IMPLEMENTED IN THE SECOND PRINTING

1. p. xviii: Amend acknowledgments to include help after the first printing (Stuck, McKay, Robbins, Walters).
2. p. 9, l. 6: Replace “ v_y ” by “ v_t ”.
3. p. 9 l. 7: Replace “ Φ_s ” by “ Φ^s ”.
4. p. 9, l. 10: Replace “ V ” by “ v ”.
5. p. 9, l. 10f: Replace “the evolution ... by ψ ” by “ $\varphi^{t+k\tau, s} = \varphi^{t, s}$ for $k \in \mathbb{Z}$ ”.
6. p. 9, l. 15f: Replace these lines by “that space is diffeomorphic to the suspension flow over the map $\varphi^{0, \tau}$ by the map $h: (\varphi^{0, t}(x), t) \mapsto (\varphi^{0, \tau}(x), t)$ ($0 \leq t \leq \tau$) to $M_{\varphi^{0, \tau}}$ ”.
7. p. 10, l. -3: Replace “ $Df_{f_x^i}$ ” by “ $Df_{f^i(x)}$ ”.
8. p. 22, l. 5: Replace “coordinates” by “coordinates (r, ϕ) ”.
9. p. 22, l. 6: Replace “const.+ log θ ” by “const. $\cdot e^{-(\theta^{-1} \log \lambda)\phi}$ ”.
10. p. 24, l. 10: Replace “under the” by “under iterates of the”.
11. p. 41, l. -1: Replace “orbits” by “points”.
12. p. 55, l. 2: Move subscripts from left side to right side of equation.
13. p. 59, l. -1: Replace “ $<$ ” by “ \leq ”.
14. p. 60, l. 4, 10: Replace “ λ^{-n} ” by “ $|\lambda|^n$ ”.
15. p. 60, l. 8: Replace “ $(n-1)!$ ” by “ $(k-1)!$ ”.
16. p. 60, l. 9: Replace “ $\binom{i}{j}$ ” by “ $\binom{l}{j}$ ”.
17. p. 60, l. 11: Replace “ λ^{-n_0} ” by “ $|\lambda|^{n_0}$ ”.
18. p. 66, l. -17: Exchange “ h ” and “ h^{-1} ”.
19. p. 68, l. -12: Delete parentheses.
20. p. 72, l. -14: Replace “map” by “ C^1 -map”.
21. p. 73, l. 8, 10, and 12: Replace “ k ” by “deg(f)”.
22. p. 74, l. 22: Replace “ $(k+1)m$ ” by “ $k(m+1)$ ” and “ a_1^i ” by “ $a_1^i \pmod{1}$ ”.
23. p. 74, l. 24: Replace “ $[\alpha_2^m, \alpha_2^{m+1}]$ ” by “ $\pi([\alpha_2^m, \alpha_2^{m+1}])$ ”.
24. p. 74, l. 29: Replace “ $a_{n-1}^{m_i}$ ” by “ $a_{n-1}^{m_i} \pmod{1}$ ”

25. p. 80, l. 5, 13; p. 81, l. -8; p. 83, l. 10; p. 86, l. 4, 7, 11, -11; p. 87, l. 6; p. 90, l. 19: Replace “ Σ ” by “ σ ”.
26. p. 80, l. -14: Replace “ $2(1 + \sqrt{2})$ ” by “ $2 + \sqrt{5}$ ”.
27. p. 80, l. -3: Replace “ $\sqrt{(\lambda^2/4) - \lambda}$ ” by “ $\sqrt{\lambda^2 - 4\lambda}$ ”.
28. p. 80, l. -2: Replace “ $\lambda|1 - 2(\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{\lambda}})|$ ” by “ $2\lambda|x - \frac{1}{2}|$ ”.
29. p. 80, l. -1: Replace “ $\sqrt{\frac{\lambda^2}{4} - \lambda} > \sqrt{(1 + \sqrt{2})^2 - 2(1 + \sqrt{2})}$ ” by “ $2\lambda\sqrt{\frac{1}{4} - \frac{1}{\lambda}} = \sqrt{\lambda^2 - 4\lambda} > \sqrt{(2 + \sqrt{5})^2 - 4(2 + \sqrt{5})}$ ”.
30. p. 81, l. -6: Replace “analysis” by “semilocal analysis”.
31. p. 90, l. 10: Replace “structurally stable” by “ C^1 strongly structurally stable”.
32. p. 102, l. 1: Replace “ $\alpha(x)$ ” by “ $a(x)$ ”.
33. p. 103, l. -10f: Replace “ $\psi^h(x)(x, 0)$ ” by “ $\psi^{h(x)}(x, 0)$ ” and “ $h(x) < \varphi(x) + h(\rho(x))$ ” by “ $h(x) < \varphi(x) + h(R_\rho(x))$ ”.
34. p. 104, l. -2: Replace “ $\alpha(\omega)$ ” by “ $a(\omega)$ ”.
35. p. 107, l. -14: Replace “a” by “at”.
36. p. 108, l. 11: Replace “ $h(f_1 x)$ ” by “ $h(\varphi^1(x))$ ”.
37. p. 109, l. 1: Replace “increase” by “decrease”.
38. p. 111, l. 4: Replace “continuous” by “uniformly continuous”.
39. p. 121, l. 5: Replace “ σ_n ” by “ σ_N ” and “-1” by “+1”.
40. p. 121, l. 10: Replace “words” by “paths”.
41. p. 121, l. 12: Replace “ n ” by “ $n + 1$ ”.
42. p. 121, l. 16: Replace “ a_{ij}^n ” by “ a_{ij}^{n+2m} ”.
43. p. 123, l. 1: Replace “ \mathbb{T}^s ” by “ \mathbb{T}^2 ”.
44. p. 123, l. 14: Replace “ $\log \frac{\sqrt{3} + 5}{2}$ ” by “ $\log \frac{3 + \sqrt{5}}{2}$ ”.
45. p. 124, l. 11: Replace “ $\max(1, \log L(f))$ ” by “ $\max(0, \log L(f))$ ”.
46. p. 125, l. 1: Replace “c, d, e” by “c, d, and e”.
47. p. 128, l. 17: Replace “ V ” by “ X ”.
48. p. 137, l. 1: Append “ $\varphi_T = \varphi_{\mathcal{I}}$ implies”.
49. p. 139, l. 3: Replace “ F ” by “ f ”.
50. p. 142, l. 10: Replace “1 =” by “ $\infty >$ ”.
51. p. 145, l. 15: Replace “ 2π ” by “ π ”.
52. p. 149, l. -12: Replace “-” by “+”.
53. p. 157, l. 9: Replace “ N for” by “ n for”.
54. p. 157, l. 17f: Replace “1” by “0” and “ N ” by “ $N - 1$ ”.
55. p. 168, l. 14: Replace “ $h \rightarrow \infty$ ” by “ $n \rightarrow \infty$ ”.
56. p. 168, l. -1: Replace “ $n = 0$ ” by “ $k = 0$ ”.
57. p. 169, l. -19: Delete “ $= \{C_\alpha \mid \alpha \in I\}$ ”.
58. p. 169, l. -17: Replace “ $\overline{\lim}_{n \rightarrow \infty} (-\log(\sup_{\alpha \in I} \mu(C_\alpha)))$ ” by “ $\overline{\lim}_{n \rightarrow \infty} -(1/n) \log(\sup_{c \in \xi_{-n}^{\mathcal{I}}} \mu(C))$ ”.
59. p. 172, l. 6: Prepend “under the factor map R ”.
60. p. 174, l. -7: Replace “ $1, \dots, N$ ” by “ $0, \dots, N - 1$ ”.
61. p. 180, l. -11f: Replace “measure preserving” by “measurable”.
62. p. 180, l. -6: Replace “and any such μ ” by “that”.
63. p. 180, l. -3: Replace “Existence of” by “For a sequence n_k with $\lim_{k \rightarrow \infty} \log \text{card}(E_{n_k}) = \overline{\lim}_{n \rightarrow \infty} \log \text{card}(E_n)$ take any accumulation point μ of the sequence μ_{n_k} . Existence of”.
64. p. 181, l. 6: Replace “ $k - 1, k + a(k)q$ ” by “ $k, k + a(k)q + 1$ ”.
65. p. 181, l. 15: Replace “ $\lim_{n \rightarrow \infty} H_{\mu_n}$ ” by “ $\lim_{k \rightarrow \infty} H_{\mu_{n_k}}$ ”.

66. p. 181, l. -2: Replace “ X , so by” by “ X . By”.
67. p. 182, l. 12: Replace “any” by “a”.
68. p. 186, l. -8 and -4; p. 187, l. 10: Replace “ $f(x)$ ” by “ x ”.
69. p. 187, l. 4: Append “.”.
70. p.188, l. 1: Replace this line by: “From now on suppose M is a compact orientable manifold. For a smooth volume form Ω we denote”
71. p. 191, l. 22: Replace “ $c_i(0) = x_i^0$ ” by “ $c_i(0) = y_i^0$ ”.
72. p. 200, l. 10: Replace “ $v_1, v_2 \in \mathbb{R}^n$ ” by “ $v_1 \neq v_2 \in \mathbb{R}^n$ ”.
73. p. 216, l. -16: Replace “ U ” by “ V ”.
74. p. 217, l. -15: Replace “ φ_{g^t} ” by “ $\varphi_{g^t}(x)$ ”.
75. p. 220, l. 17f: Replace “ $\alpha^n(v, v_2, \dots, v_n) = \dots = 0$ ” by “ $\alpha^n(v, v_2, \dots, v_n) = 0$ ”.
76. p. 220, l. 20: Replace every occurrence of “ $\alpha^{i\dots}$ ” by “ $dx_{i\dots}$ ”.
77. p. 220, l. 21: Replace “ α^1 ” by “ dx_1 ” and “ α^n ” by “ dx_{2n} ”.
78. p. 221, l. 2: Append “ $= \det(T^{-1})^t(I - \lambda T^t)$ ”.
79. p. 221, l. 3: Replace “ $A^{-1}(I - \lambda T)$ ” by “ $(I - \lambda T)T^{-1}$ ”.
80. p. 221, l. -1: Rewrite as “such that ω is in standard form at x with respect to the basis $\left\{ \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_{2n}} \right\}$ ”.
81. p. 223, l. -13, -12, -9, -8: Replace superscripts i by subscripts i .
82. p. 225, l. 3 and 6: Replace “ $\{f, \cdot\}$ ” by “ $\{\cdot, f\}$ ”.
83. p. 225, l. 7: Replace “ $\omega(X_f, X_g) = \{f, g\}$ ” by “ $\omega(X_g, X_f) = \{g, f\}$ ”.
84. p. 232, l. 10: Replace “ $\sum_{i=1}^n \frac{\partial}{\partial p_i}$ ” by “ $\sum_{i=1}^n p_i \frac{\partial}{\partial p_i}$ ”.
85. p. 233, l. 7: Replace “ $\frac{\partial}{\partial p_i} + \frac{\partial}{\partial q_i}$ ” by “ $p_i \frac{\partial}{\partial p_i} + q_i \frac{\partial}{\partial q_i}$ ”.
86. p. 239, l. 14: Delete “ f ”.
87. p. 239, l. 15: Replace “ $D_p f$ ” by “ Df_p ”.
88. p. 239, l. 23: Replace “ $D_p f^n$ ” by “ $(Df^n)_p$ ” and “ $T_f M$ ” by “ $T_p M$ ”.
89. p. 240, l. 7: Replace “ \mathbb{R} ” by “ \mathbb{R}^n ”.
90. p. 240, Theorem 6.2.3 and Definition 6.2.4: n is used both for iterates and dimension. Use m for iterates.
91. p. 240, l. 21: Replace “ \geq ” by “ $>$ ”.
92. p. 241, l. 11f: Replace “ $\text{dist}(f(y)^{-n})$ ” by “ $\text{dist}(f^{-m}(y))$ ” and “ $\text{dist}(f(y)^n)$ ” by “ $\text{dist}(f^m(y))$ ” as well as “ $n \rightarrow$ ” by “ $m \rightarrow$ ” (twice).
93. p. 242, l. 2: Insert space before “if”.
94. p. 242, l. -14: Replace “ V ” by “ $B(0, \delta)$ ”.
95. p. 243, l. -16: Replace one occurrence of “ W^+ ” by “ W^- ”.
96. p. 243, l. -9: Delete “ λ ”.
97. p. 244, l. 17: Replace “ \leq ” by “ $<$ ”.
98. p. 244, l 21f: Replace “conditions” by “assertions”.
99. p. 244, l. 23: Replace “ F ” by “ f ”.
100. p. 246, l. 6: Append “ $\cup \{0\}$ ”.
101. p. 246, l. -3, -1; p. 247, l. 2: Replace “ $(D\alpha_m)_p u$ ” by “ $(D\alpha_m)_p(u, v)$ ” and “ $(D\beta_m)_p v$ ” by “ $(D\beta_m)_p(u, v)$ ”.
102. p. 248, l. 6: Replace “linear maps” by “invertible linear maps”.
103. p. 248, l. -17f: Replace all indices n in this proof by j ; n is already in use as a dimension.
104. p. 249, l. 4: Replace “ N ” by “ \mathbb{N} ”.
105. p. 249, l. -5: Replace “ D ” by “ E ”.

106. p. 250, l. 15: Replace “Let C_γ ” by “Let C_γ^{00} ”.
107. p. 251, l. 16: Replace “ α ” by “ α ” and “ ψ ” by “ φ ”.
108. p. 254, l. 17: Replace “ $p \in$ ” by “ $p \notin$ ”.
109. p. 254ff: Rewrite Step 5.
110. p. 258f: Renumber equation.
111. p. 261, l. 16: Replace “ $h_1 = H|_S$ ” by “ $h_0 = h|_S$ ”.
112. p. 262, l. -10: Replace “ $\dim E^-(Df_p) = E^-(Dg_q)$ ” by “ $\dim E^-(Df_p) = \dim E^-(Dg_q)$ ”.
113. p. 323, l. -6: Replace “ M ” by “ m ”.
114. p. 323, l. -2: Replace “ mk ” by “ jk ”.
115. p. 324, l. 4: Replace “ m ” by “ j ” (twice).
116. p. 324, l. 5: Replace “ $n \notin \mathbb{NR}_i^f$ ” by “ $n \notin \mathbb{NR}_1^f$ ”.
117. p. 328, l. 8f: Replace “ $f_*^{(0)}$ ” by “ f_{*0} ”, “ $f_*^{(m)}$ ” by “ f_{*m} ”, and “ $f_*^{(k)}$ ” by “ f_{*k} ”.
118. p. 330, l. 17: Replace “ f_*^i ” by “ f_{*i} ”.
119. p. 336, l. -2 and -1: Replace “ $\epsilon g(x)/x^2$ ” by “ $g(x)/\epsilon x^2$ ” and “ $\epsilon(1+h(x))$ ” by “ $\epsilon x^2(1+h(x))$ ”.
120. p. 337, l. 10: Replace “from” by “form”.
121. p. 337, l. 12: Replace “to diffeomorphism” by “to a diffeomorphism”.
122. p. 337, l. 16 and 21: Add commas after “where”, “1”, and “topology”.
123. p. 338, l. 6f: Replace “By the Arzelá–Ascoli Theorem A.1.24 \mathcal{L} is compact in the uniform topology” by “ \mathcal{L} is compact in the uniform topology by the Arzelá–Ascoli Theorem A.1.24”.
124. p. 368, l. -12: Replace “maximal value of δ ” by “supremum of those δ ”.
125. p. 369, l. 9: Replace “ $\frac{d}{ds}$ ” by “ $\frac{d}{ds}|_{s=0}$ ” and “t” by “1”.
126. p. 371, l. 2 and 5: Replace “curve” by “curves” and “Approximaton” by “Approximation”.
127. p. 397, l. 10f: Replace “ $R_\tau(f)$ ” by “ $R_{\tau(f)}$ ”.
128. p. 402, l. -15: Replace “ $\varphi(| \cdot |)$ ” by “ $\varphi(\cdot)$ ” (4 times).
129. p. 402, l. 3: Replace “ $S^1 \rightarrow S^1$ ” by “ $f: S^1 \rightarrow S^1$ ”.
130. p. 402, l. -8: Replace “Next time it will come up” by “The next time it will come up is”?
131. p. 403, l. -9: Replace “point” by “points”.
132. p. 455, l. -1: Replace “defined on all of τ ” by “either defined on all of τ or not defined at all”.
133. p. 457, l. -16ff: Replace “It turns ... 14.4.3.” by “It turns out that on the Klein bottle nontrivial recurrence is impossible (see Exercise 14.2.3) and that on the torus any flow with nontrivial recurrence is equivalent to a flow under a function built from a circle diffeomorphism.”
134. p. 458, l. 1: Replace “on” by “with a nonperiodic recurrent orbit on”.
135. p. 458, l. 11 and 21: Replace “is” by “with a nonperiodic recurrent orbit is”.
136. p. 458, l. -6: Replace “flows.” by “flows, where the Poincaré Recurrence Theorem 4.1.19 provides the recurrence needed in the last two statements.”.
137. p. 460, l. 5: Delete Exercise 14.2.2.
138. p. 545, l. -2: Replace “after that proof” by “there”.
139. p. 583, l. -6: Replace “such as systems” by “such as sofic systems”.
140. p. 587, l. -9: Replace “ $\prod_{|\lambda_i| < 1} (\lambda_i^n - 1)$ ” by “ $\left| \prod_{|\lambda_i| < 1} (\lambda_i^n - 1) \right|$ ”.
141. p. 589, l. 12: Replace “ \subset ” by “ \subset ”.

142. p. 624, l. 21f: Replace “and any such μ ” by “that”.
143. p. 624, l. 24: Replace “Existence of” by “Consider a subsequence n_k such that $\lim_{k \rightarrow \infty} \log \sum_{x \in E_{n_k}} e^{S_{n_k} \varphi(x)} = \overline{\lim}_{n \rightarrow \infty} \log \sum_{x \in E_n} e^{S_n \varphi(x)}$. Take any accumulation point μ of the sequence μ_{n_k} . Existence of”.
144. p. 625, l. 9: Replace “ $\overline{\lim}$ ” by “ \lim ”, the first 5 occurrences of n by n_k , and $n \rightarrow$ by $k \rightarrow$.
145. p. 625, l. -8: Replace “any” by “a”.
146. p. 717, l. 14: Replace g_i by ψ_i and add “is nonnegative and” before “has”.
147. p. 722, l. -6f: Replace “there is a vector field v ” by “ v is a vector field”.
148. p. 723, l. 2: Replace “are” by “is that of”.
149. p. 723, l. 7, 14, 24, 27, -3; p. 724, l. 17f: Replace “ $\bigwedge^k T^*M$ ” by “ $\Gamma(\bigwedge^k T^*M)$ ” (and similar).
150. p. 723, l. 10f: Replace “If ... we” by “We”.
151. p. 724, l. 17: Delete “= f ”.
152. p. 750: Remove l. -18.
153. p. 762, l. -19: Replace “[Has1]” by “[Has]”.
154. p. 765, l. -16: Replace “ $|m|^n - 1$ ” by “ $|m^n - 1|$ ”.
155. p. 776, l. 16: Replace “3” by “2” and “map” by “map, if defined,”.
156. p. 776, l. 18: Replace “5” by “4”.
157. p. 785: Remove the reference [Has2], relabel [Has1] as [Has] and in [Has] replace “14, (1994), in press” by “14, 4, (1994), 645–666”.
158. p. 786, l. -26: Replace “1994, in press” by “79, (1994), pp. 131-156”.
159. p. 791, l. 25: Replace “4, (1983), 537–539” by “16, (1964), 61–71”.

3. CHANGES IMPLEMENTED IN THE THIRD PRINTING

1. p. 42, l. 1f: Change to: “Consider the family f_λ .
 - (a) Calculate $P_n(f_4)$.
 - (b) Calculate $P_n(f_3)$.
 - (c) Show that for $\lambda > 3$ there is a period-two orbit.
 - (d) Show that for $\lambda \in (3, 1 + \sqrt{6}]$ there are no points of period higher than two.
2. p.52, l. 17: Change “operator” to “theorem”.
3. p. 63, l. -8: Replace “ $f_{3.05}$ and $f_{3.1}$ ” by “Show that for $\lambda_1, \lambda_2 \in (3, 1 + \sqrt{6})$ the maps f_{λ_1} and f_{λ_2} ”.
4. p. 79, l. -10: Replace “ f^n ” by “ f^{-n} ”.
5. p. 80, l. 12: Insert “For it to be a semiconjugacy, however, we had to deviate slightly from the prescription (2.5.1) because the sets Δ_n^k in the proof of Proposition 1.7.2 (or the sets Δ_n^m in (2.4.5)) are not of the form $\bigcap_{i=0}^n f^{-i} \Delta_1^{\omega_i}$, for example, $E_2^{-1}(\Delta_1^0) \cap \Delta_1^0 = \Delta_2^0 \cup \{1/2\}$. Thus, instead of (2.5.1), we consider expressions like

$$\bigcap_{n \in \mathbb{Z}} \overline{\text{int} \left(\bigcap_{|k| \leq n} f^{-k}(X_{\omega_k}) \right)} \quad (2.5.2)$$

to ensure that despite overlaps on the boundaries we obtain intersections consisting of a single point.”

6. p. 80, l. 13: Replace “ Σ ” by “ σ ”.
7. p. 81, l. -1: Replace “ Σ ” by “ σ ”.
8. p. 83, l. 10: Replace “ Σ ” by “ σ ”.

9. p. 86, l. 4, 7, 11, 24: Replace “ Σ ” by “ σ ”.
10. p. 86, l. 14: Replace “(2.5.4)” by “(2.5.5)”.
11. p. 103, l. -10: Replace “ $\psi^h(x)(x, 0)$ ” by “ $\psi^h(x)(x, 0)$ ”.
12. p. 108, l. -12: Replace “ j ” by “ i ”.
13. p. 109, l. -9: Replace “less than” by “at most”.
14. p. 111, l. 4: Replace “continuous” by “uniformly continuous”.
15. p. 111, l. -1: Replace “ d_{nm}^f ” by “ d_{nm-m+1}^f ”.
16. p. 112, l. 1: Replace “any $d_n^{f^m}$ ϵ -ball contains a d_{mn}^f ϵ -ball” by “ $B_f(x, \epsilon, mn - m + 1) \subset B_{f^m}(x, \epsilon, n)$ for any $x \in X$ ”.
17. p. 136, bottom half: Shorten proof.
18. p. 137, l. 1: Append “ $\varphi_T = \varphi_{\mathcal{I}}$ implies”.
19. p. 142, l. 10: Replace “ $1 =$ ” by “ $\infty >$ ”.
20. p. 144, l. -2: Replace “ σ_2 ” by “ σ_2^k ”.
21. p. 145, l. 3: Replace “(4.1.11)” by “(4.1.10)”.
22. p. 145, l. 15: Replace “ 2π ” by “ π ”.
23. p. 184, l. -1: Replace “ f -invariant” by “ T -invariant”.
24. p. 185, l. 5: Replace “ f ” by “ T ”.
25. p. 195, l. 6f: Replace “the Poincaré Lemma (Theorem A.3.11)” by “Lemma A.3.13”.
26. p. 199, l. 15: Replace “ $\frac{A^2}{Cx_1 + Dx_2}$ ” by “ $\frac{rA^2}{r + Cx_1 + Dx_2}$ ” and “ $\frac{A^2}{Cr \cos \alpha + Dr \sin \alpha}$ ” by “ $\frac{A^2}{1 + C \cos \alpha + D \sin \alpha}$ ”.
27. p. 205, l. -12: Replace “flow” by “flow (Definition 5.3.4)”.
28. p. 250, l. 15: Replace “ $C_\gamma(\mathbb{R}^k)$ ” by “ $C_\gamma^0(\mathbb{R}^k)$ ”.
29. p. 274, l. -6: Replace “ n ” by “ k ”.
30. p. 282, l. 10: Replace “4” by “6”.
31. p. 312, l. 10f: Replace “the de Rham Theorem” by “Lemma A.3.13”.
32. p. 341, l. 6: Replace “cos” by “ $-\cos$ ”.
33. p. 343, l. 1: Replace “(9.2.2)” by “(9.2.1)”.
34. p. 351, l. 3f: Replace “ f ” by “ h ” and “ F ” by “ f ”.
35. p. 404, l. -12: Replace “ $d(x, y)$ ” by “ $d(x, y)^\alpha$ ”.
36. p. 489, l. -9f: Replace “ K ” by “ L ” twice, “ $d =$ ” by “ $d :=$ ” and “ $c' =$ ” by “ $c' :=$ ”.
37. p. 490, l. 14: Replace “ $\leq n$ ” by “ $< n$ ”.
38. p. 492, l. 18: Replace “ A ” by “2”.
39. p. 500, l. 14: Replace “ $\triangleleft 7$ ” by “ $\triangleleft 9 \triangleleft 7$ ”.
40. p. 501, l. 9: Replace “ $y > a$ ” by “ $y > b$ ”.
41. p. 501, l. -9ff: Replace “Moreover, we have observed that the left endpoint of I_{2k} is mapped to the right endpoint of I_{2k+1} for all k . In particular $f(x_{\min}) = x_{\max}$ and, since I_{p-1} covers I_1 , we have $I_{p-1} \rightarrow I_k$ for all odd k .” by “Writing $a_i := f^i(a)$ we have shown $x_{\min} = a_{p-1} < a_{p-3} < \dots < a - 2 < a < a_1 < a_3 < \dots < a_{p-2} = x_{\max}$, hence evidently $I_{p-1} = [a_{p-1}, a_{p-3}] \rightarrow I_k$ for odd k .”.
42. p. 502, l. 15f: Rewrite as: “Our assumption implies that $[x_{\min}, a]$ does not cover I_1 , so $f([x_{\min}, a])$ is to the right of a .”
43. p. 503, l. 21: Replace “By Theorem 15.1.9” by “By Theorem 15.1.9 and the Perron–Frobenius Theorem 1.9.11”.

44. p. 504, l. 11: Replace “ $\mathcal{P}(f) := \{n \in \mathbb{N} \mid \text{Fix}(f^n) \neq \emptyset\}$ ” by “ $\mathcal{P}(f) := \{n \in \mathbb{N} \mid \text{Fix}(f^n) \setminus \bigcup_{k < n} \text{Fix}(f^k) \neq \emptyset\}$ ”.
45. p. 504, l. 22: Replace “a homeomorphism and thus has periodic point of period at most two” by “a linear expanding homeomorphism and thus has a unique fixed point and no other periodic points”.
46. p. 504, l. -8: Replace “length” by “odd length”.
47. p. 536, l. 7: Replace “ $\{h(\omega)\} = \bigcap_{i \in \mathbb{Z}} f^{-i}(\Lambda_{\omega_i})$ ” by “ $\{h(\omega)\} = \bigcap_{n \in \mathbb{Z}} \overline{\text{int}(\bigcap_{|k| \leq n} f^{-k}(\Lambda_{\omega_k}))}$ ”.
48. p. 536, l. 8: Append “analogously to (2.5.2)”.
49. p. 587, l. 16ff: Replace “The assertion is equivalent to hyperbolicity of the linear map f_* induced by f on the fundamental group \mathbb{Z}^n of \mathbb{T}^n because the associated matrix defines F_L on \mathbb{T}^n . First we use the Spectral Decomposition Theorem 18.3.1 to write the nonwandering set $NW(f)$ as a union of components on which some power f^N is topologically mixing. Consider also the unstable bundle E^+ of f . After passing to a double cover of \mathbb{T}^n (which is still a torus) we may assume that E^+ is orientable in the sense explained in Exercise 8.6.3. Furthermore on each mixing component f^N consistently either preserves or reverses the orientation of E^+ , so passing to f^{2N} we may assume that f is topologically mixing on $NW(f)$ and preserves the orientation of E^+ . In this case all periodic points of a given period have the same index, either 1 or -1 (Corollary 8.4.7). Thus by Corollary 8.6.16 of the Lefschetz Fixed-Point Formula and (8.7.1) which gives the Lefschetz number of a toral map we have” by “This is equivalent to hyperbolicity of the linear map f_* induced by f on the fundamental group \mathbb{Z}^n of \mathbb{T}^n because the associated matrix defines F_L on \mathbb{T}^n . By the Spectral Decomposition Theorem 18.3.1 the nonwandering set $NW(f)$ is a union of components on which some power f^N is topologically mixing, so $P_n(f^N)$ has a multiplicative exponential asymptotic by Theorem 18.5.6. After passing to a double cover of \mathbb{T}^n (which is still a torus) we may assume that the unstable bundle E^+ of f is orientable in the sense explained in Exercise 8.6.4. Then on each mixing component f^N consistently either preserves or reverses the orientation of E^+ , so passing to f^{2N} we may assume that f preserves the orientation of E^+ (and $P_n(f)$ has a multiplicative exponential asymptotic). Then all periodic points of a given period have the same index, either 1 or -1 (Corollary 8.4.7). Thus by Corollary 8.6.16 of the Lefschetz Fixed-Point Formula and (8.7.1) which gives the Lefschetz number of a toral map we have”.
50. p. 587, l. -2f: Last sentence reads: “This contradicts existence of an exponential multiplicative asymptotic of $P_n(f)$.”
51. p. 601, l. 20: Replace “Theorem” by “Theorem 6.2.8”.
52. p. 601, l. 28: Replace “an” by “a $C^{1+\beta}$ ”.
53. p. 602, l. -7: Replace “ $< \epsilon$ ” by “ $\leq L\|z\|^\beta < \epsilon$ ”.
54. p. 603, l. 7: Replace “ $\leq (\lambda - \epsilon) + \epsilon = \lambda$ ” by “ $\leq (\lambda - \epsilon)\|E_z\| + l\|z\|^\beta$ ”.
55. p. 603, l. 9: Replace “ $\mu^{-1}\lambda$ ” by “ $\mu^{-1}(\lambda - \epsilon)\|E_z\| + l\|z\|^\beta$ ”.
56. p. 603, l. 14: Replace “then for any $K > 0$ ” by “and $K \geq L/\epsilon$ then $\mu^{-1}((\lambda - \epsilon)K + L) \leq \mu^{-1}\lambda K$ and hence”.
57. p. 638, l. 8: Replace “2e” by “1e”.
58. p. 660, l. -4: Replace “a” by “an invertible”.
59. p. 661, l. -1: Replace “ n ” by “ x ”.
60. p. 662, l. -6: Replace “ $\chi > 0$ ” by “ χ ”.

61. p. 668, l. 5: Replace “ $\chi_i - \epsilon$ ” by “ $\chi_i(x) - \epsilon$ ”.
62. p. 671, l. 5: Replace “ $C \in$ ” by “ $C \in \xi$ ”.
63. p. 671, l. 14: Replace “ r'^n ” by “ r^n ”.
64. p. 671, l. 15: Replace “ $(r')^n$ ” by “ $(r')^{mn}$ ”.
65. p. 676, l. 10: Replace “ W ” by “ V ”.
66. p. 703, l. 8: Replace “ $\bigcup_{\alpha \in A}$ ” by “ $\bigcup_{\alpha \in A} O_\alpha$ ”.
67. p. 725, l. 14: Add “We shall have occasion to invoke a result related to the preceding ones:
Lemma A.3.13. On an n -manifold an n -form with zero integral is exact.”
68. p. 742, l. 12f: Replace “will appear in a forthcoming book” by “is given in the book [LM]”.
69. p. 743, l. 2: Add “There are a number of books on low-dimensional dynamics, including texts (for example [De], [Be]) as well as monographs ([Me], [MeS], [CE]). From among the rapidly growing literature attempting to use low-dimensional dynamics and connections to applications to a wider audience we particularly recommend that by Strogatz [Str] for its lively and intelligent treatment of applications.”
70. p. 743, l. 20: Add “Variational methods are a very important tool in Hamiltonian dynamics. A recent account of these is in [FM].”
71. p. 743, l. 24: Add “Symplectic topology and its relations to Hamiltonian dynamics are presented in the recent book [HZ] by Hofer and Zehnder, which starts at a basic level and explains connections between a certain rigidity of symplectic maps and periodic phenomena in dynamics in terms of symplectic capacities.”
72. p. 750, l. -18: Delete.
73. p. 752, l. 20: Add: “The first few bifurcations in the quadratic family are the subject of Exercise 1.7.2, they occur for $\lambda = 3$ and $\lambda = 1 + \sqrt{6}$. Via algebra, which is still straightforward in principle, but quite formidable, one can show that for $\lambda = 1 + \sqrt{8}$ a 3-periodic point first appears (this is mentioned, with references both to the original and an elementary proof, in [Str]). By Theorem 15.3.2 the entire periodic-doubling cascade must therefore end somewhat earlier (and, moreover, orbits of all periods must have appeared by then).”
74. p. 755, l. -8: Add “This chapter is dominated by variational methods. The variational approach can be applied to Hamiltonian systems in a very similar way. For a recent exposition see [FM].”
75. p. 784, l. 15: Add “[FM] Forni, Giovanni, and Mather, John: Action minimizing orbits in Hamiltonian systems. Transition to chaos in classical and quantum mechanics, edited by S. Graffi, Lecture Notes in Mathematics, vol. 1589, pp. 92–186, Springer Verlag, Berlin, Heidelberg, New York, 1994.”
76. p. 785, l. -17: Add “[HZ] Hofer, Helmut, and Zehnder, Eduard: Symplectic invariants and Hamiltonian dynamics, Birkhäuser, Basel, Boston, 1994.”
77. p. 792, l. 13: Add “[Str] Strogatz, Steven H.: Nonlinear dynamics and chaos, Addison–Wesley, Reading, MA, 1994.”

4. CHANGES IMPLEMENTED IN THE FOURTH PRINTING

1. p. 30, l. -10: Replace “argue by contradiction” by “prove the contrapositive”.
2. p. 31, l. 4–8: Rewrite as “which means that either $\chi_{k_1, \dots, k_n} = 0$ or $\exp(2\pi i \sum k_i \gamma_i) = 1$, that is, $\sum k_i \gamma_i$ is an integer. Since both U and its complement contain

- nonempty open sets, which have positive Lebesgue measure, χ is not constant almost everywhere. Therefore there is some $(k_1, \dots, k_n) \neq 0$ such that $\chi_{k_1, \dots, k_n} \neq 0$ and hence $\sum k_i \gamma_i$ is an integer.”
3. p. 37, l. -7–1: Rewrite as “Since M is compact, the set $\omega_F(x)$ is nonempty. Suppose $\omega_F(x)$ is finite and $y, z \in \omega_F(x)$, $y \neq z$. We have $y = \lim \varphi^{t_n}(x)$ and $z = \lim \varphi^{s_n}(x)$, where as before $\{\varphi^t\}$ is the gradient flow. Let B be a ball around y and S the boundary of B such that $(B \cup S) \cap \omega_f(x) = \{y\}$. Since the orbit of x enters and leaves B infinitely many times the intersection $\mathcal{O}(x) \cap S$ is an infinite set and by compactness of S it contains a limit point which must belong to $\omega(x)$.”
 4. p. 59f: This proof rewritten to represent more accurately the majorization method.
 5. p. 83, l. 17: Replace “ f^n ” by “ f^{-n} ”.
 6. p. 131, l. 4: Replace “ F ” by “ f ”.
 7. p. 136, l. 13: Replace “ f ” by “ T ”.
 8. p. 138, l. 16: Replace “ $L_1(X, \varphi)$ ” by “ $L^1(X, \mu)$ ”.
 9. p. 142, l. 2: Replace “Lebesgue” by “probability”.
 10. p. 142, l. 10: Replace “ $\infty >$ ” by “ $1 =$ ”.
 11. p. 143, l. -11: Delete “Lebesgue”.
 12. p. 145, l. 1: Replace “Lebesgue” by “measure”.
 13. p. 148, l. -2: Replace “ L_1 ” by “ L^1 ”.
 14. p. 149, l. -10: Replace “metric!isomorphism” by “metric isomorphism”.
 15. p. 150, l. -2: Replace “ L_1 ” by “ L^1 ”.
 16. p. 152, l. -6, -4, -2, p. 153, l. 14, 15: Replace “ L_2 ” by “ L^2 ”.
 17. p. 154, l. 6: Replace “ \mathbb{T}^n ” by “ \mathbb{T}^m ”.
 18. p. 155, l. 4: Replace “ L_1 ” by “ L^1 ”.
 19. p. 155, l. 6: Replace “would follow” by “follows”.
 20. p. 156, l. 15f: Delete “Assume that p has at least two nonzero components. Then the shifts” and “, respectively”.
 21. p. 157, l. 19: Replace “for” by “to”.
 22. p. 161, l. -1: Replace “ $(A \cup B) \setminus (A \cap B)$ ” by “ $(A \cup B) \setminus (A \cap B)$ ”.
 23. p. 163, l. 9: Delete “the Lebesgue space”.
 24. p. 191, l. 15: Replace “ Γ_n^m ” by “ Γ_m^n ”.
 25. p. 192, l. 3–5: Replace by “Inequalities as in this lemma are called *bounded distortion estimates* and will appear several times in the study of one-dimensional systems, both invertible and not. This inequality could be used in place of the C^2 hypothesis of Theorem 5.1.16.”
 26. p. 233: Add a section on algebraic dynamics.
 27. p. 239, l. -5, -2, p. 240, l. 5: Replace “ φ_t ” by “ φ^t ”.
 28. p. 243: Some streamlining of the statement by incorporating the remarks below.
 29. p. 250, l. 15: Replace “ \subset ” by “ \in ”.
 30. p. 260, l. -11: Replace “ $W^+(0)$ ” by “ $W^-(0)$ ”.
 31. p. 265, l. -3: Replace “ N ” by “ V ”.
 32. p. 269, l. 17: Replace “ $\text{dist}(f(x_0), x_0)$ ” by “ $\text{dist}(f^m(x_0), x_0)$ ”.
 33. p. 271, l. -3: Replace “ k ” by “ n ”.
 34. p. 274, l. -6: Replace “ f^k ” by “ f^{-k} ”.
 35. p. 357, l. -5: Replace “ $\tilde{\Sigma}$ ” by “ Σ ”.
 36. p. 485, l. 19: Replace “ $(n-1)$ th” by “ $(n-1)$ st”.

37. p. 501, l. 8: Replace “ \mathcal{C} ” by “ $\mathcal{C} \setminus \{I_1\}$ ”.
38. p. 501, l. -10: Replace “ f ” by “4”.
39. p. 501, l. -9: Replace “ $a - 2$ ” by “ a_2 ”.
40. p. 502, l. 17: Replace “ $[b, x_{\max}]$ ” by “ $f([b, x_{\max}])$ ”.
41. p. 504, l. 8: Delete “other”.
42. p. 536, l. 7: Replace “ k ” by “ i ”.
43. p. 536, l. 10: Replace “ \cap ” by “ $\text{int } \cap$ ”.
44. p. 538, l. -2: Replace “Note that it” by “ $\Lambda := \mathbb{T}^2 \setminus W$ ”.
45. p. 540, l. -7: Replace “17.2.1” by “17.2.2.”
46. p. 540, l. -3: Replace “ s ” by “ S ”.
47. p. 545, l. 2: Replace “ φ_t ” by “ φ^t ”.
48. p. 562, l. -5: Insert a subsection on holomorphic dynamics.
49. p. 566, l. -18: Replace “diffeomorphism” by “ C^2 diffeomorphism”.
50. p. 567, l. -6: Replace “ α^{-1} ” by “ g^{-1} ”.
51. p. 617, l. -2: Replace “ \subset ” by “ \in ”.
52. p. 619, l. 4: Replace “ \subset ” by “ \in ”.
53. p. 738: Add about a page on locally compact groups, Haar measure, lattices.
54. p. 745, l. 2: Replace “[P]” by “[P1]”.
55. p. 745, l. 22: Replace “The expected answer is known as the ‘Williams Conjecture’ [Wi2], [Boy].” by “Williams [Wi2], [Boy] suggested an answer known as the ‘Williams Conjecture’. Attempts to prove it produced numerous fruitful developments in symbolic dynamics concluding, somewhat anticlimactically, in a recent counterexample by Kim and Roush.”
56. p. 745, l. -10: Replace “The main estimates of this proof were provided by David DeLatte.” by “This proof was shown to us by Jürgen Moser.”
57. p. 749, l. 8: Replace “.” by “and the book by Parry [P2].”
58. p. 753, l. 7: Add “Kaloshin showed nongenericity [Kal]: There is an open set U of C^k diffeomorphisms such that for any sequence $\{a_n\}$ there is a residual subset of U where $P_n(f)/a_n \rightarrow \infty$.”
59. p. 757, l. 8: Replace “a forthcoming paper of Gutkin and Katok” by “[GuK]”.
60. p. 759, l. 1: Replace “The original proof of Theorem 15.3.2 is” by “Theorem 15.3.2 and the first half of Theorem 15.3.7 were first proved”.
61. p. 759, l. 20: Replace “Świątek who built on joint work with Jacobson as well as work by Sullivan and others.” by “Graczyk and Świątek [GraSw] who built on work by Sullivan, Świątek and Jacobson, and others.”
62. p. 761, l. -7: Replace “Mañé” by “Mañé, Lyubich, Milnor”.
63. p. 764, l. -9: Add “This section reproduces arguments of C. Toll [To].”
64. p. 766, l. 3: Replace by “**1.9.11.** Set $a_{04} = a_{15} = a_{24} = a_{35} = a_{42} = a_{43} = a_{50} = a_{51} = 1$, all other entries zero, define H by $0 \mapsto 00, 1 \mapsto 01, 2 \mapsto 10, 3 \mapsto 11, 4 \mapsto 01, 5 \mapsto 10$. Only the orbit of $\dots 101010\dots$ has more than one preimage.”
65. p. 784, l. 3: Add “[Do] Dolgopyat, Dmitry: On decay of correlations in Anosov flows. *Annals of Mathematics* **147** (1998) no. 2, 357–390 [DoP] Dolgopyat, Dmitry and Pollicott, Mark: Addendum to ‘Periodic orbits and dynamical spectra’ (by Viviane Baladi). *Ergodic Theory And Dynamical Systems* **18** (1998) no. 2, 255–292”

66. p. 784, l. -3: Add “[GraSw] Graczyk, Jacek, Świątek, Grzegorz: Generic hyperbolicity in the logistic family. *Annals of Mathematics* **146** (1997) no. 1, 1–52”
67. p. 785, l. 11: Add “[GuK] Gutkin, Eugene, and Katok, Anatole: Caustics in inner and outer billiards. *Communications in Mathematical Physics* **173** (1995), 101–133”
68. p. 789, l. -2: Replace “[P]” by “[P1]”.
69. p. 789, l. -0: Add “[P2] Parry, William: Entropy and generators in ergodic theory: W. A. Benjamin, Inc., New York–Amsterdam, 1969”
70. p. 791, l. 32: Add “English translation: *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering* **5** (1995) no. 5, 1263–1273
On cycles and structure of a continuous map: *Ukrainskiĭ Matematicheskiĭ Zhurnal* **17** (1965) no. 3, 104–111